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Role of the Clogging Phenomena in Erroneous Implications of Conventional Data Analysis for Constant Pressure Cake Filtration

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ABSTRACT

Constant pressure filtration of kaolin slurries is studied by directly measuring the pressure drop across the filter medium under conditions leading to various extents of medium clogging. Experimental data are analyzed in terms of the conventional filtration theory based on the two-resistances-in-series model. The filter medium resistance and the cake resistance values obtained by the conventional data interpretation techniques, which require only the measurement of total filtration pressure, are compared with those calculated from directly measured pressure drops across the filter medium and the cake. The results show that the conventional analysis yields unrealistic filter medium resistances if the medium clogging is significant and continues throughout the course of filtration. The medium resistance is a significant part of the total filtration resistance even at long filtration times. The average specific cake resistance obtained by the conventional methodology does not appear to be in error when compared to its value calculated from the directly measured cake resistance at long filtration times. An actually incompressible cake, however, appears to be compressible due to cake clogging which increases with increasing filtration pressure.

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INTRODUCTION

Conventional theory of cake filtration is based on a two-resistances-in-series (the filter medium resistance and the cake resistance) model. The following instantaneous reciprocal filtrate rate equation of the conventional theory has served as a basis for interpreting laboratory tests and for developing design equations for constant-pressure (vacuum), constant-rate, and centrifugal-pump filtrations (1):

$$\frac{dt}{dV} = \frac{1}{Aq} = \frac{\mu\langle\alpha\rangle c}{A^2 P} V + \frac{\mu R_m}{AP} \tag{1}$$

where t = filtration time, V = cumulative filtrate volume, A = filter area, q = instantaneous superficial filtrate rate, μ = filtrate viscosity, $\langle\alpha\rangle$ = average specific cake resistance (cake resistance per unit mass of dry solids in the cake deposited per unit area), c = mass of solids filtered per unit volume of filtrate, P = applied filtration pressure, and R_m = filter medium resistance. Equation (1) has been expressed in resistance form ($P/\mu q$) to facilitate comprehension of the physical phenomena represented by it:

$$R = \frac{P}{\mu q} = \left(\langle\alpha\rangle \frac{c}{A}\right) V + R_m = R_c + R_m \tag{2}$$

where R = instantaneous total filtration resistance and R_c = cake resistance given by the product of $\langle\alpha\rangle$ and the mass of dry cake solids per unit area (cV/A).

The conventional theory has traditionally assumed that medium resistance R_m is constant and the average cake resistance $\langle\alpha\rangle$ depends primarily on the particulate material and the pressure drop across the cake. If the parameters $\langle\alpha\rangle$ and c (or their product) are also constant during the course of filtration, Eq. (2) leads to linear plots of $P/\mu q$ vs V data for constant pressure filtration. The average specific cake resistance, $\langle\alpha\rangle$, is determined from the slope of the linear plot, while the intercept gives the supposedly constant medium resistance, R_m. If $\langle\alpha\rangle$ increases and the average cake porosity decreases with increasing filtration pressure, then the filter cake is said to be compressible. This is the conventional data analysis technique, or the so-called textbook approach (2), which has been firmly entrenched in the filtration literature.

When fine particulates of a slurry are deposited in the interstices of the filter cake or medium during the course of filtration (cake and medium clogging), basic assumptions involving the constancy of R_m and $\langle\alpha\rangle$ are violated (3). Notebaert et al. (4) observed deviations from linear plots of the total resistance versus filtrate volume, which were explained by the phenomena of cake and medium clogging. They proposed that the relative importance of the clogging of the cake and the medium determines the shape of the total resistance versus filtrate volume plots, and that it is not impossible to obtain linear plots if both phenomena compensate each other (i.e., R_m increases at a decreasing rate, while R_c rises at an increasing rate). This suggests that the value of $\langle\alpha\rangle$ obtained from the slope of the conventional resistance plot may be at least partially dependent upon the selected filter medium. In fact, Notebaert et al. (4) considered the resistance of the medium together with the resistance of the filter cake as a whole, probably to avoid this dependency of $\langle\alpha\rangle$ on R_m.

Tiller and coworkers (3, 5–7) also recognized that traditional data interpretation techniques were frequently in error due to incompatibility of the basic assumptions with experimental data, and proposed a revised interpretation of instantaneous total resistance plots. According to the revised data analysis methodology, $P/\mu q$ vs V plots may have curved regions depending upon assumptions concerning the magnitude of R_m and the variation of R_m and $\langle\alpha\rangle$ during the entire filtration due to medium and cake clogging, which may lead to false R_m values (even negative values in the case of severe cake clogging). They also pointed to the impossibility of obtaining medium and cake resistances from a single overall pressure drop in the case of variable R_m and suggested that the liquid pressure at the filter medium should be measured to avoid the potentially hazardous assumption of constant filter medium resistance.

An alternative cake filtration theory based on the volume-averaged multiphase continuum theory (8, 9) postulated that the greatest part of the total filtration resistance will develop at the cake–medium contact where gradual blocking and clogging of the pores of the filter medium take place. The filter medium coated with particles trapped in its pores is said to dictate the filtrate rate and govern the filtration behavior of slurries with changes brought about at the cake–medium contact. Based on this mechanism, Willis (9) indicated that the $\langle\alpha\rangle$ parameter of the conventional two-resistances model must be dependent not only on the properties of the filter cake but also on the selected filter medium. Willis et al. (10) and Tosun (11) also deduced from theoretical considerations that the intercept of the $P/\mu q$ vs V plot is not related to R_m but it is simply the initial resistance of the clean medium. Chase et al. (12) evaluated the interactions of the filter medium with the filter cake from the continuum theory perspective, and found out that the medium resistance increased with cake height. Chase and Willis (13) pointed out that an actually incompressible cake may appear as compressible just because of the presence of air bubbles constricting the flow channels through the cake near the medium.

The majority of investigators still prefer to use the standard textbook data analysis techniques of filtration based on the conventional theory in spite of its serious deficiencies, especially those with regard to variations in R_m. Misinterpretations are very likely to arise if the simplifying assumptions upon which the conventional methodology depends are not critically verified by experimental filtration data. The present article attempts to extend the range of the available data on the important role of the filter medium and cake clogging phenomena in data analysis and analyzes data from constant pressure filtration experiments conducted with different particulate solids–filter medium combinations by using a filtration apparatus equipped with a pressure probe at the cake–medium interface. The objectives are 1) to compare the filter medium resistance and the average specific cake resistance values obtained from the conventional data analysis techniques with those experimentally determined from pressure readings at the cake–medium interface, and 2) to test if the average specific cake resistance determined by conventional data analysis is affected by the extent of medium clogging.

EXPERIMENTAL

An industrially ground kaolin clay sample (density = 2.61 g/cm^3) was separated by hydrocyclone treatment into two fractions of different particle size distributions, and the two fractions, named as "coarse" sample (median size = 19 μm; 80% finer than 25 μm, 4% finer than 3 μm) and "fine" sample (median size = 5 μm; 96% finer than 25 μm, 30% finer than 3 μm), were used in filtration tests. Filtration slurries were prepared by suspending either of these test samples in distilled water so as to obtain the desired solids concentration and filtered through two types of filter papers: a coarse porosity (25 μm) filter paper (Toyo No. 5A) and a fine porosity (0.2 μm) cellulose acetate-type membrane (Sartorius SM 11107), referred to as "coarse" medium and "fine" medium, respectively.

Experiments were run with a filtration setup having a downward-filtering, pressure-type, Plexiglas filter chamber of 10.8 cm diameter and 10.8 cm height (91.61 cm^2 filter area and 989.4 cm^3 volume). The filter chamber was initially filled with clear water from a wash tank. The filtration slurry, maintained as an homogeneous suspension by stirring in a pressurized slurry tank, was then introduced into the filter chamber, and the filtration pressure was immediately adjusted to a predetermined constant value by manually adjusting a needle valve fitted to the slurry tank. The clock was started at the same instant as the pressure adjustment, and this was nominally time zero for filtration. Since the filter chamber was initially filled with clear water, it was difficult to assign the effective time of the start of actual filtration. For time–filtrate volume data analysis the effective onset of filtration was set to 10 seconds of the nominal filtration time and the filtrate volume data were corrected accordingly for the amount of filtrate (around one-third of the filter chamber volume) collected during this very short initial period. Total filtration time was 3 hours for the fine sample slurry filtration runs, and 13 minutes for the coarse sample–coarse medium and 30 minutes for the coarse sample–fine medium filtration runs.

The filtrate volume was obtained from the weight of the filtrate collected in a beaker placed on the electronic balance, and the cake thickness was measured visually with a scale on the filter chamber. The average cake porosity was determined by measuring the wet and dry filter cake mass at the end of filtration. The instantaneous superficial filtrate rate, q, was obtained from the instantaneous slope of the cumulative filtrate volume vs time data. No smoothing of the data was attempted.

The liquid pressure at the cake–filter medium interface was measured with a vertical glass probe of 2.5 mm inside diameter, placed about 0.4 mm above the filter medium. The probe was connected to a pressure dial gauge having a range of 0–100 kPa with $\pm$0.5% accuracy. All tubing and connections of the pressure sensing system were flushed with water prior to each experiment to ensure no air was entrained in the system. As the bottom of the filter medium support plate was open to atmospheric pressure and the resistance of the plate was assumed negligible due to its perforated design, the local liquid gauge pressure immediately above the medium gave the pressure drop across the filter medium, ΔP_{m}. The pressure drop across the filter cake, ΔP_{c}, was obtained by difference from the constant filtration pressure. The medium resistance, R_{m}, and the cake resistance, R_{c}, were then calculated from the relationships

$$R_{\mathrm{m}} = \Delta P_{\mathrm{m}}/\mu q \qquad (3)$$

$$R_{\mathrm{c}} = \Delta P_{\mathrm{c}}/\mu q \qquad (4)$$

while assuming a constant superficial liquid velocity at any instant throughout the cake. Noteworthy is the fact that the medium resistance measured as such is a composite value comprising the resistance of the filter medium itself, the pores of which are being clogged as well as blocked by migrating fine particles, and the resistance of the particle layers in 0.4 mm of filter cake adjacent to the medium. One cannot experimentally separate the two latter resistances. Furthermore, it is the interaction of the filter medium with the particulates of the filter cake at the medium–cake contact which determines the extent of medium clogging or blocking. Therefore, we consider the composite resistance as the effective filter medium resistance, and will simply call it the medium resistance.

Constant-pressure filtration experiments were planned so that the interactions of the coarse and fine particulate samples with the coarse and fine media could be studied in terms of the extent of filter medium clogging at the constant solids concentration of 5% by weight in suspension (c = 52.63 kg of solids per m^3 of filtrate) at 30 kPa filtration pressure. Additional experiments were run at 60 and 90 kPa pressures with the fine medium–fine sample filtration system to study the effect of pressure on the extent of medium clogging and its possibly deceptive implication in filtration data analysis.

EXPERIMENTAL RESULTS AND DISCUSSION

Experimental results indicating the effect of the selection of the filter medium on filtration resistances of a given particulate sample are shown in Figs. 1 and 2 for the coarse and fine particulate samples, respectively. The experimental data are in the two sets of two resistance plots in each figure. Each set represents filtration runs with one of the two types of filter medium used in the study. The two resistance plots are the instantaneous total filtration resistance (R) and the filter medium resistance (R_m) calculated from direct measurements of the liquid pressure at the cake–medium interface. The clean filter media resistances, as calculated from clear-water permeation tests conducted at 30 kPa pressure with the clean media, are shown in Fig. 1 as horizontal dashed lines. The effect of filtration pressure on the resistances R_m and R for the fine sample filtered with the fine filter medium is shown in Figs. 3 and 4, respectively. The variation of measured cake resistance (R_c) with fil-

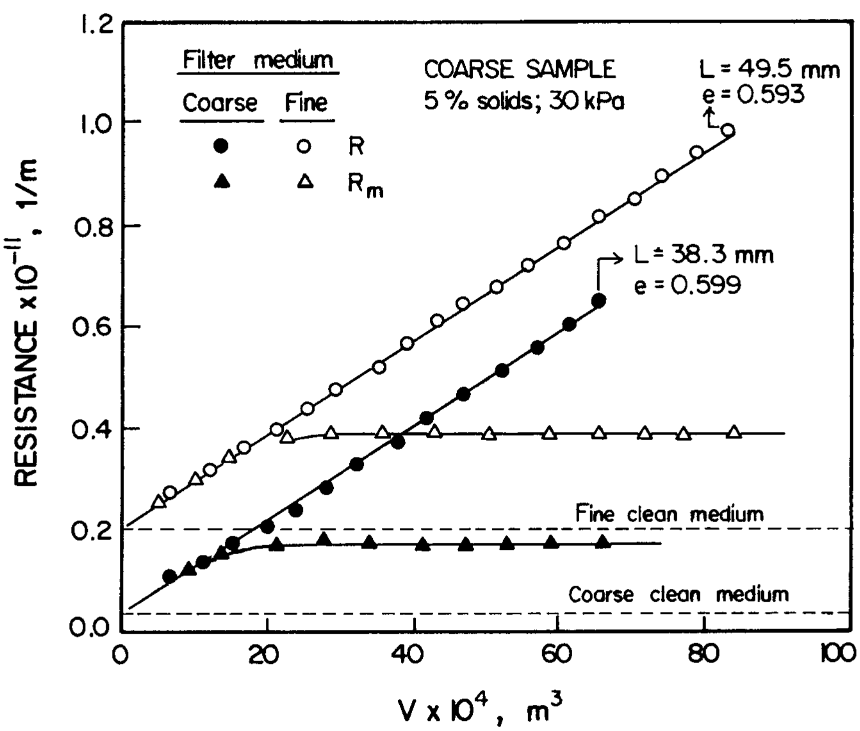

FIG. 1 Plots of total filtration resistance (R) and measured medium resistance (R_m) versus cumulative filtrate volume (V) for the coarse particulate sample.

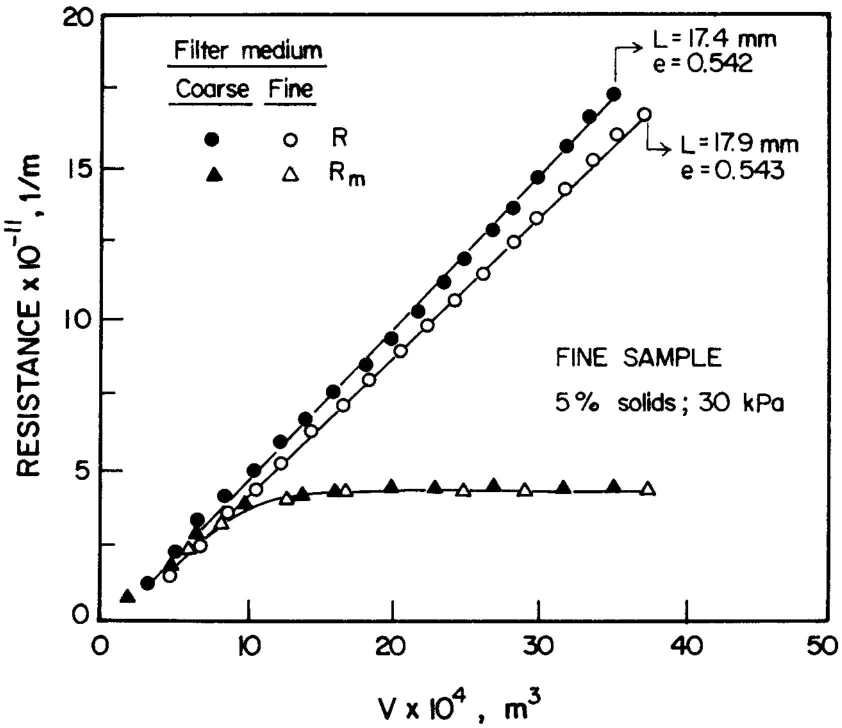

FIG. 2 Plots of total filtration resistance (R) and measured medium resistance (R_m) versus cumulative filtrate volume (V) for the fine particulate sample.

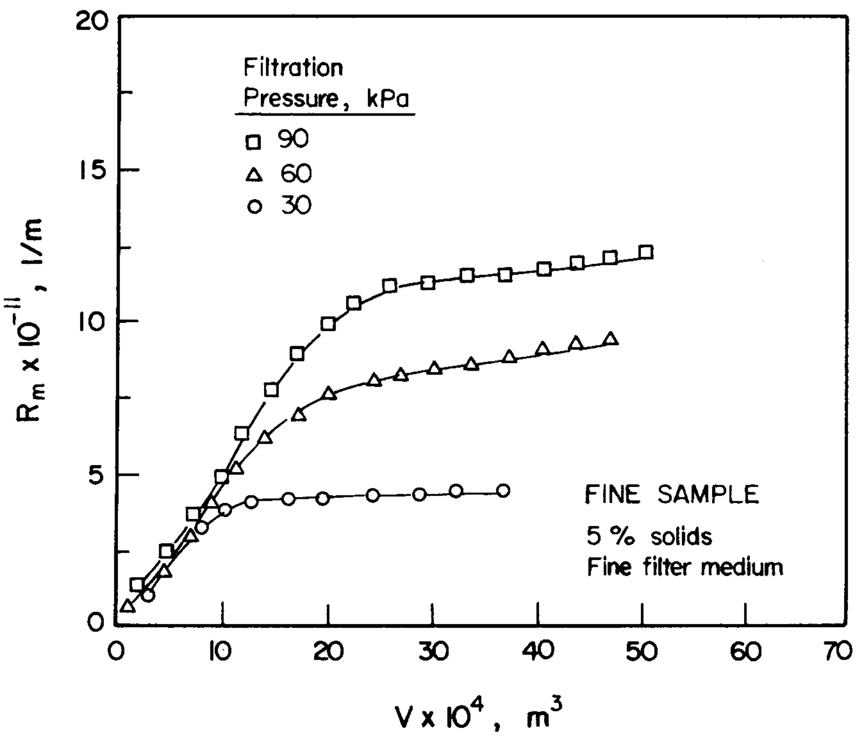

FIG. 3 Plots of measured medium resistance (R_m) versus cumulative filtrate volume (V) for the fine particulate sample at three different filtration pressures.

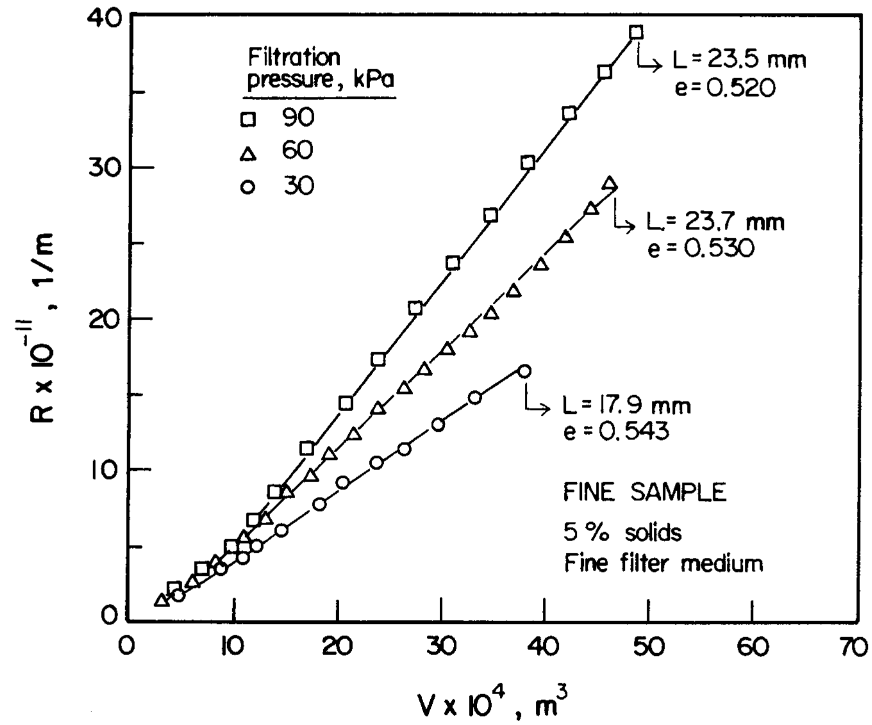

FIG. 4 Conventional total resistance plots for three different filtration pressures.

trate volume is shown in Fig. 5. The visually measured cake thickness (L) versus filtrate volume (V) data for the fine particulate sample are presented in Fig. 6.

In all cases, directly measured R_m values increase rapidly at the start of filtration while particles of the same size as or larger than the pores of the medium wedge into the openings and some finer particles inevitably penetrate into the pores (medium clogging). Although no quantitative monitoring of filtrate turbidity was accomplished, cloudy filtrates observed at the start of the filtration runs were indicative of medium clogging. This initial rapid rise of R_m extends to greater filtrate volumes with increased filtration pressure (Fig. 3). Fine particles were presumably able to migrate through thicker cakes into the medium as a result of higher filtrate rates at higher pressures.

As the filter cake builds up, R_m approaches its ultimate value at a rate specific to filtration conditions. R_m values for the filtration runs performed at 30 kPa remain practically unchanged in the latter stages of filtration, whereas R_m values for the 60- and 90-kPa runs keep increasing gradually after the initial rapid rise period. The ultimate values of R_m for the coarse particulate sample (Fig. 1) are not drastically larger than the clean medium resistances for 30-kPa filtration runs, only 3.8 times as much for the coarse medium and 1.6 times as

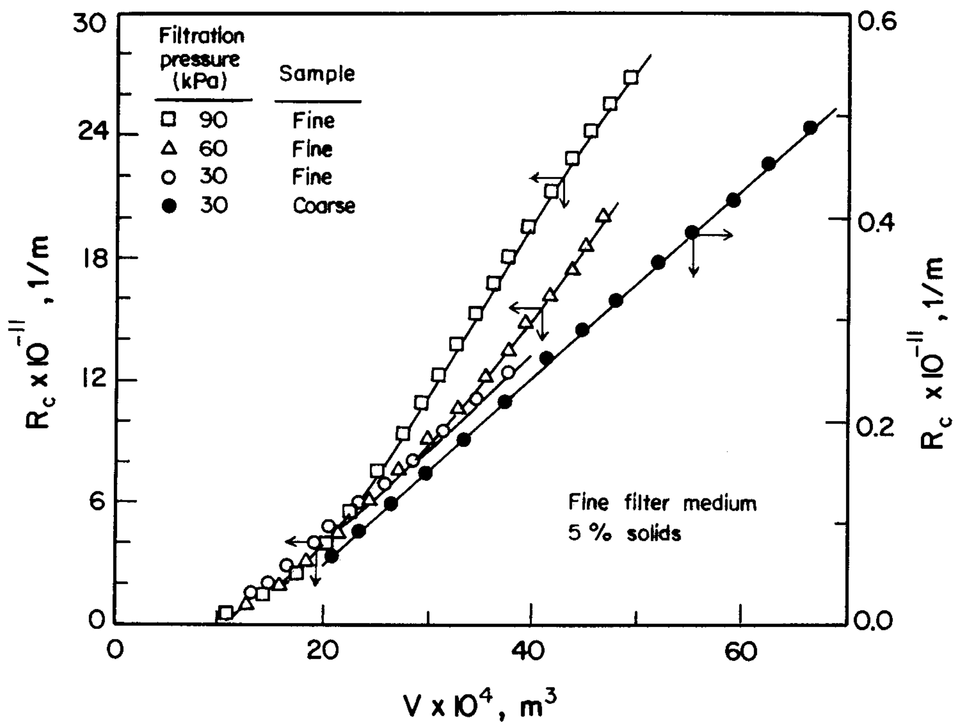

FIG. 5 Plots of measured cake resistance (R_c) versus cumulative filtrate volume (V).

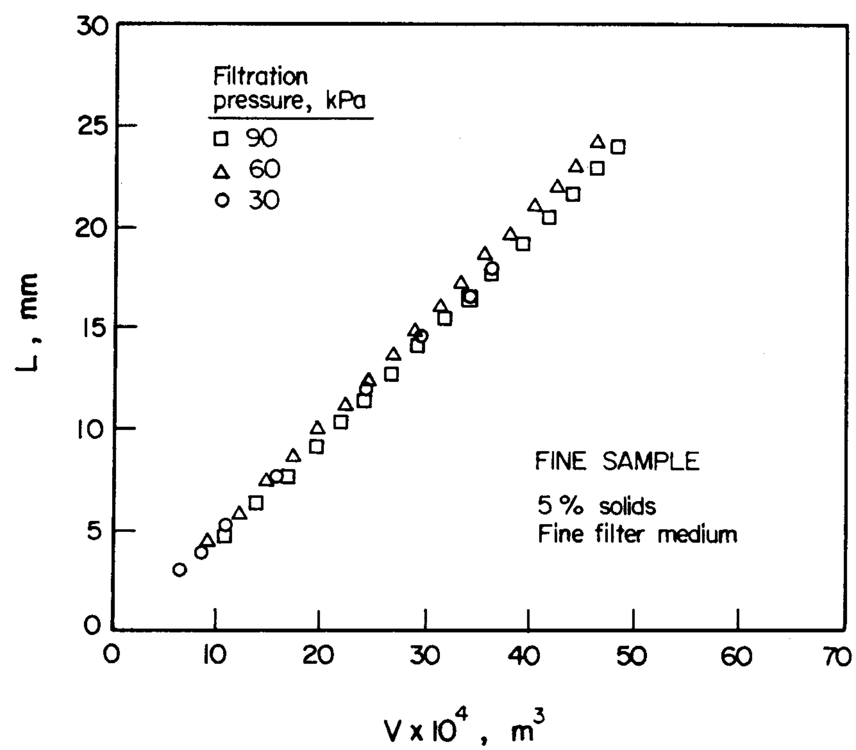

FIG. 6 Plots of cake thickness (L) versus cumulative filtrate volume (V).

much for the fine medium. The fine particulate sample and coarse medium have the greatest increase in medium resistance of about 80 times for 30 kPa filtration pressure (Fig. 2). The fine particulate sample and fine medium have an increased ultimate R_m by a factor of about 20 to 60 times, the increase being more significant under higher filtration pressure (Fig. 3).

It is of interest to compare the relative magnitudes of R_m, R_c, and the total resistance R during the filtration runs. At the initial stages of filtration, the filtration resistance is mainly due to R_m, and then the contribution from R_c increases as the cake builds up. The R_c versus V plots in Fig. 5 appear to be linear for that portion of the filtration where R_m increases slowly or remains unchanged (cf. Fig. 3). In the case of the fine particulate sample, R_m remains dominant over R_c until a cake of 10 to 15 mm thickness has formed (the cake thickness for a certain V can be obtained from Fig. 6). The corresponding cake thicknesses for the coarse particulate sample are almost twice as much. Contrary to the presumed filtration mechanism of the conventional theory, R_m never becomes less important than R_c, and constitutes at least about 25 to 30% of the total filtration resistance even at the end of a filtration run.

The instantaneous total resistance plots of the present study (Figs. 1, 2, and 4) yield linear lines except for slight curvature in the initial portion, the curvature being more pronounced with the filtration runs in which R_m increases throughout the course of filtration (60- and 90-kPa runs in Figs. 3 and 4). Analysis of the initial nonlinearity, which spans a very short period of the total filtration time, is complicated by two inseparable effects: one is the fact that the filter chamber is initially filled with clear water, and it takes a while before a steady solids concentration and actual filtration conditions are attained; the other one is that this is the period during which R_m undergoes a rapid rise while clogging and blocking of the medium take place. Owing to the design of the filtration apparatus, one cannot precisely determine the true origin for collecting the time and filtrate volume data. The slope of the instantaneous R versus V plot is, however, not affected by the uncertain origin of t versus V data.

The medium resistances inferred from the extrapolated intercepts of the conventional resistance plots of the coarse particulate sample (Fig. 1) are of approximately the same magnitude as the corresponding clean medium resistances. The extrapolated intercepts of the linear regions of the plots in Fig. 4, however, yield physically meaningless negative R_m values, although the measured R_m values for the same runs shown in Fig. 3 are significantly large. This phenomenon may not be explicable solely by the uncertain origin of filtrate volume. In an earlier study using a filter assembly initially filled with suspension, rather than clear water, Bridger et al. (14) presented plots similar to those in Fig. 4, and attributed such negative medium resistances to medium blinding. They also suggested that the value of R at the transition point from the initial region to the second linear region should be considered an apparent medium resistance. The transition point for their data corresponded to a cake thickness of 1.5 mm. If we were to apply their suggestion to our plot for 90 kPa in Fig. 4, we would obtain an apparent R_m value of 5×10^{11} m^{-1} at $V =$ 1000 cm^3 (cake thickness L, 5 mm; Fig. 6). This apparent R_m is almost one-third of the measured ultimate R_m value (ultimate L is 23.5 mm), and it is quite unreasonable to add the resistance of 5 mm-thick cake to the resistance of the filter medium. This flaw in the conventional data interpretation is more likely to arise from its unrealistically assumed filtration mechanism which considers the filter medium resistance an unchanging part of the total filtration resistance.

Values of the average specific cake resistance, $\langle\alpha\rangle$, obtained by the graphically determined slope of the latter linear portion of the conventional R vs V plots and by that of the R_c vs V plots, are given in Table 1, together with the average cake porosity values at the end of each filtration. The $\langle\alpha\rangle$ values obtained as such for each filtration run agree quite well, particularly so for the runs at 30 kPa filtration pressure, in which R_m is nearly constant after a rapid initial change. The gradual increase in R_m during the course of filtrations at 60 and 90 kPa does not appear to cause significant differences between the $\langle\alpha\rangle$ values obtained by the two methods. The average cake porosity for a given particulate sample is independent of the type of filter medium at 30 kPa filtrations. There is, however, an 85% increase in $\langle\alpha\rangle$ for the fine particulate sample when the applied filtration pressure increases from 30 to 90 kPa, accompanied by a small, perceptible decrease in the cake porosity from 0.543 to 0.520.

The classical theory would normally categorize filter cakes as compressible or compactible if the specific cake resistance increases with increasing filtra-

TABLE 1
Average Cake Porosity and the Average Specific Cake Resistance $\langle\alpha\rangle$ Calculated by Use of Both the Conventional Resistance Plots (R vs V) and the Measured Cake Resistance Plots (R_c vs V)

Particulate sample	Filter medium	Filtration pressure (kPa)	Average cake porosity	$\langle\alpha\rangle \times 10^{-10}$ m/kg	
				R vs V	R_c vs V
Coarse	Coarse	30	0.599	0.16	0.16
Coarse	Fine	30	0.593	0.16	0.16
Fine	Coarse	30	0.542	8.77	8.73
Fine	Fine	30	0.543	8.36	7.90
Fine	Fine	60	0.530	11.64	12.35
Fine	Fine	90	0.520	15.52	14.62

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tion pressure. An increase in ⟨α⟩ would result from a decrease in the average cake porosity. According to the Carman–Kozeny equation, ⟨α⟩ should be directly proportional to a porosity function of the form $(1 - e)/e^3$ (15, 16). Applying this proportionality to the aforementioned difference in the cake porosity values at two different filtration pressures would predict an increase of about 20% in ⟨α⟩ as opposed to the measured increase of 85%. Furthermore, as porosity decreases with increasing pressure for a compressible cake, the average cake porosity is expected to decrease during the course of constant-pressure filtration such that a plot of L vs V would curve toward the V axis. The linearity of plots exhibited in Fig. 6, however, suggests that the average porosity is either truly constant during the course of filtration or its variation is too small compared to the precision of the measurements. It appears that the filtration pressure range of this work is not high enough to see any significant compressibility of the kaolin filter cakes during the course of filtration.

Inasmuch as there is continued medium clogging, it is logical to assume that the cake also is subject to clogging. The most likely location within the cake for clogging to occur is the initial layers adjacent to the medium, which will result in a denser, less porous region dictating filtration rates. The porosity distribution across the cake may be very similar irrespective of the filtration pressure except for this compact region next to the medium, which seems to be the most likely cause of the slightly reduced average cake porosity with increasing pressure. Hence, the increase in ⟨α⟩ by a factor disproportionate to the decrease in the average cake porosity is thought to be explicable by the denser deposition of migrating fines in the rate-limiting compact region at the bottom of the cake.

Another point worth mentioning is the linearity of the conventional resistance plots (Fig. 4) despite the presence of clogging, which is best examplified by our experimental data from the 60-kPa run with the fine sample. The R_c profile for this filtration run in Fig. 5 exhibits an increase in R_c at a slowly increasing rate, while R_m for the same run in Fig. 3 increases at a slowly decreasing rate. The two effects apparently compensate for each other, yielding a linear total resistance profile which is in agreement with Notebaert's aforementioned compensation effect (4).

CONCLUSIONS

The analysis of constant pressure filtration experimental data by using instantaneous total filtration resistance plots of the conventional two-resistances filtration theory yields unrealistic filter medium resistances in the presence of medium clogging during the course of filtration. The direct measurements indicate that the filter medium resistance constitutes a substantial portion (25 to 30%) of the overall filtration resistance even at the end of filtration runs with reasonably thick (about 18 to 50 mm) filter cakes. In spite of the gradual in-

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crease in the medium resistance during the course of filtration, the average specific cake resistance $\langle\alpha\rangle$ obtained from the slope of the conventional total resistance plot does not appear to be erroneous when compared to the correct value of $\langle\alpha\rangle$ which is obtained from the measured cake resistance at the end of long-time filtrations. Using $\langle\alpha\rangle$ as a sole measure of filterability, however, should be undertaken very cautiously since the medium resistance R_m is a significant part of the total filtration resistance. The clogging of the filter medium and, hence, its resistance is dependent of the applied filtration pressure as well as the relative size of the filter medium pores for a given size of particulates in the slurry. The increase in $\langle\alpha\rangle$ with increasing filtration pressure is attributed largely to the cake clogging phenomenon rather than cake compressibility.

NOTATION

A	filter area (m^2)
c	mass of solids filtered per unit volume of filtrate (kg/m^3)
e	average cake porosity (dimensionless)
L	cake thickness (m)
P	applied filtration pressure (N/m^2)
ΔP_c	pressure drop across the filter cake (N/m^2)
ΔP_m	pressure drop across the filter medium (N/m^2)
q	instantaneous superficial filtrate rate (m/s)
R	total filtration resistance (1/m)
R_c	cake resistance (1/m)
R_m	filter medium resistance (1/m)
t	filtration time (s)
V	cumulative filtrate volume (m^3)
$\langle\alpha\rangle$	average specific cake resistance (m/kg)
μ	filtrate viscosity ($N\cdot s/m^2$)

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